Assignment 2 Solutions

September 19, 2025

Problems

1. Find the limit (if it exists). If it does not exist, explain why:

$$\lim_{x\to 3} f(x), \qquad f(x)=\begin{cases} x^2-4x+6, & x<3,\\ -x^2+4x-2, & x\geq 3\,. \end{cases}$$

2. Find the constant a such that the function is continuous on $\mathbb{R}$:

$$g(x)=\begin{cases} \dfrac{4\sin x}{x}, & x<0,\\[2ex] a-2x, & x\geq 0\,. \end{cases}$$

3. Find the one-sided limit (if it exists):

$$\lim_{x\to 3^+}\left(\frac{x}{3}+\cot\frac{\pi x}{2}\right).$$

Solutions and Explanations

1. Limit of a piecewise function at $x=3$

Compute one-sided limits:

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3}(x^2-4x+6)=9-12+6=3.$$

$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3}(-x^2+4x-2)=-9+12-2=1.$$

Since $3\neq 1$, the two one-sided limits are different, hence

$$\boxed{\lim_{x\to 3} f(x) \text{ does not exist.}}$$

(附註：由定義，$f(3)=-3^2+4\cdot 3-2=1$，也因此在 $x=3$ 不連續。)

2. Continuity of $g(x)$ on $\mathbb{R}$

連續性只需在拼接點 $x=0$ 檢查。對 $x<0$，

$$\lim_{x\to 0^-}\frac{4\sin x}{x}=4\cdot\lim_{x\to 0^-}\frac{\sin x}{x}=4.$$

對 $x\geq 0$ 的分段，$g(0)=a-2\cdot 0=a$，且 $\lim_{x\to 0^+}(a-2x)=a$。令左、右極限與函數值相等得 $a=4$。因此

$$\boxed{a=4 \text{ 時，} g(x) \text{ 在全實數連續.}}$$

3. One-sided limit at $x=3^+$

因為 $x=3$ 不是 $\sin\left(\frac{\pi x}{2}\right)=0$ 的點（$\frac{\pi x}{2}=k\pi\Rightarrow x=2k$），$\cot\left(\frac{\pi x}{2}\right)$ 在 $x=3$ 連續且

$$\cot\left(\frac{\pi\cdot 3}{2}\right)=\cot\left(\frac{3\pi}{2}\right)=0.$$

因此

$$\lim_{x\to 3^+}\left(\frac{x}{3}+\cot\frac{\pi x}{2}\right)=\frac{3}{3}+0=\boxed{1}.$$

Final Answers: (1) DNE (left $=3$, right $=1$); (2) $a=4$; (3) 1.